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A New Semiclassical Elementary Particle Model

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Abstract

A semiclassical model in which elementary particles are represented as systems of charged shells with associated quark-like quantum numbers is presented. Specifically the baryons are considered. Formulas are obtained which express baryon masses and magnetic moments in terms of model parameters which relate baryon and quark properties. Basically, the mass and moment formulas are expressions for mass ratios and magnetic moment ratios. Simple identifications for the model parameters lead to a prediction for the proton–electron mass ratio and to fairly accurate predictions for the baryon magnetic moments in units of the proton moment.

The mass and moment formulas, which relate corresponding properties of different particles, are generalised such as to express relationships between the members of a sequence of particles, where such a sequence is conceived of as containing only one (normal) baryon. A specific sequence, containing the proton and electron, is proposed; various physical properties of the particles in the sequence are determined. In particular, a second prediction for the proton–electron mass ratio is obtained; the two predictions differ numerically but both agree with the measured value of the mass ratio within experimental error.

1. *Introduction*

This paper presents the latest results of a continuing investigation whose objective is to find a simple semiclassical model of elementary particles capable of providing accurate estimates of particle properties (masses and magnetic moments). The present model represents particles as systems of charged shells as was done in some previously considered models (Delaney, 1973). Here a new parameterisation of particle properties is formulated. Within this formalism fairly accurate estimates of some particle properties are obtained in a rather simple way.

As in the previously considered models, the particle subshells may be of different types, a shell type being defined by a set of quantum numbers. Three shell-types, denoted by the symbols p, n, and λ are considered; their associated quantum numbers are displayed in Table 1. Since the shell-type quantum numbers correspond to those of the p, n, and λ quarks, a shell of

TABLE 1. Shell types p, n and λ and their associated quantum numbers. The quantum numbers are identical with the charge, hypercharge, isospin, strangeness and baryon number quantum numbers originally associated with p, n, and λ quarks

	q	Y	I	I	S	B
p	2/3	1/3	1/2	1/2	0	1/3
n	−1/3	1/3	1/2	−1/2	0	1/3
λ	−1/3	−2/3	0	0	−1	1/3

type p, n, or λ will be considered synonymous with a p, n, or λ quark. In general, a particle will be identified in terms of its quantum numbers, which are the sums of the quantum numbers of its constituent shells or quarks.

The formal representation of the models to be considered is based on the classical relation†

$$W = \int d^3r(E^2 + B^2)/8\pi \tag{1.1}$$

expressing the rest frame electromagnetic energy, W, associated with a particle in terms of the particle's electric, $\mathbf{E}$, and magnetic, $\mathbf{B}$, fields. The field $\mathbf{E}$ is the sum of the electric fields $\mathbf{E}_i$ due to the shell charges $q_i e$ (e is the charge of the proton); thus, denoting the shell radii by R_i,

$$\mathbf{E} = q_i e\mathbf{r}/r^3 \qquad r > R_i \tag{1.2a}$$

$$\mathbf{E} = 0 \qquad r < R_i \tag{1.2b}$$

The field $\mathbf{B}$ is the sum of the magnetic fields $\mathbf{B}_i$ arising from magnetic dipole moments, $\boldsymbol{\mu}_i$, associated with the shells or quarks. The fields $\mathbf{B}_i$ are assumed to have the form which would result if the moments were due to rotation of charged shells; thus

$$\mathbf{B}_i = 3(\boldsymbol{\mu}_i \cdot \mathbf{r})/r^5 - \boldsymbol{\mu}_i/r^3 \qquad r > R_i \tag{1.3a}$$

$$\mathbf{B}_i = 2\boldsymbol{\mu}_i/R_i^3 \qquad r < R_i \tag{1.3b}$$

In this paper only the baryons will be considered explicitly. As usual, these particles will be built from three quarks; in the context of the present model they will correspond to systems of three shells.

2. *The Model*

The quark magnetic moments are defined in terms of their g-factors, g_i, masses, m_i, and angular momenta, $\mathbf{J}_i$, as

$$\boldsymbol{\mu}_i = e(g_i/2m_i)q_i\mathbf{J}_i \tag{2.1}$$

† Gaussian units, and the natural units obtained by setting $\hbar = c = 1$, are used throughout this paper.

Resulting relationships will be simplified by assuming that two of the quark shells have various common properties; specifically, either

$$R_2 = R_3, \qquad g_2 = g_3, \qquad m_2 = m_3 \tag{2.2a}$$

or

$$R_2 = R_1, \qquad g_2 = g_1, \qquad m_2 = m_1 \tag{2.2b}$$

where, by convention,

$$R_1 \leqslant R_3 \tag{2.3}$$

In either of the cases (2.2a) or (2.2b), the result of evaluating (1.1), using (1.2), (1.3), (2.1) and (2.3), may be expressed in the form

$$2/e^2 = [Q_{00} + 2(g_1/2m_1R_1)^2K_{00}^2]/WR_1 + [Q_0 + 2(g_3/2m_3R_3)^2K_0^{\,2}]/WR_3 \tag{2.4}$$

where, defining

$$\epsilon = (g_1/g_3) \qquad m_3/m_1 \tag{2.5}$$

in the case (2.2a),

$$Q_{00} = q_1^{\,2} \tag{2.6a}$$

$$Q_0 = (q_1 + q_2 + q_3)^2 - q_1^{\,2} \tag{2.6b}$$

$$K_{00}^2 = (q_1J_1)^2 \tag{2.6c}$$

$$K_0^{\,2} = K^2 - \epsilon^2K_{00}^2 \tag{2.6d}$$

where

$$\mathbf{K} = q_3\mathbf{J}_3 + q_2\mathbf{J}_2 + \epsilon q_1\mathbf{J}_1 \tag{2.6e}$$

while, in the case (2.2b)

$$Q_{00} = (q_1 + q_2)^2 \tag{2.7a}$$

$$Q_0 = (q_1 + q_2 + q_3)^2 - (q_1 + q_2)^2 \tag{2.7b}$$

$$K_{00}^2 = (q_1\mathbf{J}_1 + q_2\mathbf{J}_2)^2 \tag{2.7c}$$

$$K_0^{\,2} = K^2 - \epsilon^2K_{00}^2 \tag{2.7d}$$

where

$$\mathbf{K} = q_3\mathbf{J}_3 + \epsilon(q_2\mathbf{J}_2 + q_1\mathbf{J}_1) \tag{2.7e}$$

The expression (2.4) may be formally transformed into an expression involving the mass, M, of a particle (quark system) by the following procedure. Quantities b_i, relating the quark-shell masses to their radii, are defined by

$$m_iR_i = b_i \tag{2.8}$$

Similarly, a quantity d, relating the radius, R, of a particle to its electromagnetic self-energy, is defined by

$$WR = d \tag{2.9a}$$

since obviously

$$R_3 = R \tag{2.9b}$$

$$WR_3 = d \tag{2.9c}$$

$$m_3 R = b_3 \tag{2.9d}$$

Using (2.8) and (2.9) with (2.4) and defining

$$N_1 = Q_{00} + 2(g_1/2b_1)^2 K_{00}^2 \tag{2.10a}$$

$$N_3 = Q_0 + 2(g_3/2b_3)^2 K_0{}^2 \tag{2.10b}$$

the expression

$$m_1/m_3 = (2d/e^2 - N_3)b_1/(b_3 N_1) \tag{2.11}$$

is obtained for the quark mass ratio. Defining a parameter, α, relating the mass, M, of a particle to the mass ratio of its constituent quarks by

$$M/m = \alpha m_1/m_3 \tag{2.12}$$

where m is the electron mass (the significance of this relation will be investigated in the following section), and using (2.11),

$$M/m = (2d/e^2 - N_3)b_1\alpha/(b_3 N_1) \tag{2.13}$$

Assuming that the quark masses, m_i, are the same for all particles (a major specific difference between this and the previous models), (2.12) may be used to relate the fixed quark mass ratio to the properties of one baryon. Choosing this baryon to be the proton and establishing the convention of distinguishing properties associated with the proton (including those of its constituent quarks) by a '(P)',

$$m_1/m_3 = (1/\alpha)M/m = (1/\alpha(P))M(P)/m \tag{2.14}$$

Using (2.14) with (2.11),

$$M(P)/m = (2d/e^2 - N_3)b_1\alpha(P)/(N_1 b_3) \tag{2.15}$$

Assuming that the magnetic moment of a particle, $\boldsymbol{\mu}$, is the sum of the moments of its constituent quarks,

$$\boldsymbol{\mu} = e(g_3/2m_3)\mathbf{K} \tag{2.16}$$

where $\mathbf{K}$ is given by either (2.6e) or (2.7e). Assuming again that m_3 is the same for all baryons, the moments of these particles may be expressed in terms of proton properties as

$$\mu/\mu(P) = g_3 K/(g_3(P)K(P)) \tag{2.17}$$

Since, with (2.14), the quantity ϵ defined by (2.5) will be very small (the order of $m/M(P)$), the quantities K in (2.17) can be replaced to great accuracy by the corresponding values of K_0 (defined by (2.6d) or (2.7d)); thus

$$\mu/\mu(P) = g_3 K_0/(g_3(P) K_0(P)) \tag{2.18}$$

In order to obtain precise predictions from (2.18) and (2.15) it is necessary to identify constraints whereby, for the various baryons, the values of the parameters appearing in these expressions may be determined or at least restricted to a limited set of possible values.

The set of quarks corresponding to each baryon is uniquely determined from the condition that its various quantum numbers must equal the sums of the corresponding quantum numbers of its constituent quarks. This condition fixes the possible shell charge assignments for each baryon, there remaining, however, the choice of which quarks correspond to which shells, that is, the identification of the type (p, n, or λ) of each shell.

The quark angular momenta and their couplings are restricted by the condition that the sum of the quark angular momenta must equal the baryon spin, **S**. This condition implies that

$$S^2 = 3/4 = J_{12}^2 + J_{13}^2 + J_{23}^2 - J_1{}^2 - J_2{}^2 - J_3{}^2$$

where the $J_{ik}^2 = (\mathbf{J}_i + \mathbf{J}_k)^2$ are squares of the 'intermediate angular momenta'. The values of the J_{ik}^2 will be used to specify the quark angular momentum coupling; these quantities may be conveniently employed for the evaluation of the expressions (2.7c), (2.7d) and (2.6d) which (with (2.6c)) determine the values of the quantities K_0 and K_{00} in (2.10).

The quark angular momenta will also be limited by allowing only the value

$$j_i = 1/2 \tag{2.19a}$$

for their total angular momentum quantum number, in which case

$$J_i^2 = j_i(j_i + 1) = 3/4 \tag{2.19b}$$

and the $(J_{12}^2, J_{13}^2, J_{23}^2)$ can only assume values corresponding to some permutation of either $(3/2, 3/2, 0)$ or $(2, 1/2, 1/2)$.

The values of the d and b_i for the various baryons are not determined by any obvious *a priori* physical considerations. However the form of (2.15) suggests some simple assumptions whereby the choice of these quantities (and the angular momentum couplings) may be limited.

Imposing the condition that (2.15) be linear in $M(P)/m$ (that it yield a unique value for this ratio), then, given (2.19) and the possible quark charges, the quark angular momentum couplings can only correspond: in the case (2.2a) to

$$(J_{12}^2, J_{13}^2, J_{23}^2) = (3/2, 3/2, 0) \tag{2.20a}$$

and in the case (2.2b) to

$$(J_{12}^2, J_{13}^2, J_{23}^2) = (0, 3/2, 3/2) \tag{2.20b}$$

since, in this way, the dependence of the quantities K_0 in (2.10b) on the ratio $m/M(P)$ (through (2.6e), (2.6d), (2.7e), (2.7d), (2.5)) and (2.12) is eliminated. With (2.19) and (2.20), in the case (2.2a),

$$(K_0/S)^2 = (q_2 - q_3)^2 \tag{2.21a}$$

while in the case (2.2b),

$$(K_0/S)^2 = q_3{}^2 \tag{2.21b}$$

Introducing also the formal assumptions (whose physical significance will be discussed later)

$$g_i = g_i(P) \tag{2.22a}$$

$$d = d(P) \tag{2.22b}$$

$$N_3 = N_3(P) \tag{2.22c}$$

$$b_1/(N_1 b_3) = b_1(P)/(N_1(P)b_3(P)) \tag{2.22d}$$

then, once the values of the parameters associated with the proton have been fixed, the b_i and the magnetic moment ratios (2.18) for each of the other baryons will depend only on the choice of (2.2a) or (2.2b) and on the specific shell type assignments assumed.

Assuming for the proton: the case (2.2a) (and thus (2.20a)), the identifications (p, p, n) for its shells $(1, 2, 3)$ and the parameter values

$$d(P) = 1 \tag{2.23a}$$

$$b(P) = 1 \tag{2.23b}$$

$$g(P) = 2 \tag{2.23c}$$

$$\alpha(P) = 15/2 \tag{2.23d}$$

and using (2.15),

$$M(P)/m = (1/e^2 - 37/36)27/2 = 1836{\cdot}1113 \tag{2.23e}$$

$$M(P) = 938{\cdot}2604 \text{ MeV} \tag{2.23f}$$

where $1/e^2 = 137{\cdot}03602$ and $m = {\cdot}5110041$ MeV have been used. This value for the proton mass is in good agreement with the measured value, $938{\cdot}2592 \pm {\cdot}0052$ MeV. The values of m, e^2, $M(P)$ and the error in $M(P)$ have been taken from Particle Data Group (1973). For subsequent reference, the values of $K_0(P)$ and $K(P)$ are given by

$$K_0(P)/S = 1 \tag{2.23g}$$

$$(K(P)/S) \cong 1{\cdot}000004 \tag{2.23h}$$

With (2.19) and (2.23a) the magnetic moment ratios (2.18) are given by

$$\mu/\mu(P) = K_0/K_0(P) = K_0/S \tag{2.24}$$

Table 2 displays the values of the magnetic moment ratios obtained from (2.24). The choice of (2.2a) or (2.2b) (and thus of (2.20a) or (2.20b)) is indicated for each baryon by enclosing the pair of quarks corresponding to shells of equal radius in parentheses. The values obtained for the b_i using (2.10) with the assumed proton parameters (2.23a–c) are also shown.

The tabulated magnetic moment ratios are equivalent to well-known predictions of the non-relativistic quark model and are consistent with the $SU(6)$ result $\mu(N)/\mu(P) = -2/3$ and the $SU(3)$ relations $\mu(\Sigma^+) = \mu(P)$, $\mu(\Xi^-) = \mu(\Sigma^-)$ and $\mu(N) = \mu(\Xi^0)$. These values for the magnetic moment ratios are only in approximate agreement with experiment, the result for the Λ being particularly inaccurate.

Having assumed $j_i = 1/2$, the identifications $g_i = 2$ for the quark g-factors are consistent with the assumption that quarks are simple Dirac particles, whose orbital angular momenta, for baryons, is zero.

TABLE 2. The baryon magnetic moments in units of the proton moment from (2.24), and the b_i obtained using (2.22c, d). Parentheses indicate which two quark symbols label the particle subshells having equal radii according to (2.2a, b)

	Quarks (1, 2, 3)	$\lvert\mu/\mu(P)\rvert$	$1/b_3^2$	b_3/b_1
Σ^+	$p(p\,n)$	1	1	1
Σ^-	$(n\,n)\lambda$	1/3	9	5/2
Ξ^-	$(\lambda\,\lambda)n$	1/3	9	5/2
N	$(n\,n)p$	2/3	15/4	5/2
Ξ^0	$(\lambda\,\lambda)p$	2/3	15/4	5/2
Σ^0	$(p\,n)\lambda$	1/3	13	$\sim\sqrt{(7)}/7$
Λ	$(p\,n)\lambda$	1/3	13	$\sim\sqrt{(7)}/7$

The assumption that $d = 1$ for all baryons corresponds to the effective elimination of this parameter and the assumption that the radius and the electromagnetic self-energy of all baryons are related in general by $WR = 1$. The assumption that also $b_3(P) = 1$ corresponds according to (2.9a, d) to the assumption that the quark mass m_3 equals the electromagnetic self-energy of the proton, $m_3 = W(P)$.

From a physical point of view, the formal conditions (2.22c, d) would correspond to the assumption that the fraction of the total electromagnetic energy of a baryon due to its inner (and thus, also outer) shell (or shells, depending on (2.2a, b) is the same for all baryons, this interpretation relying on (2.22b) and on the assumption that m_1/m_3 is constant.

3. *Generalisation to Particle Sequences*

In this section an attempt is made to elucidate the possible physical significance of the various parameters appearing in (2.18) and (2.15) and, in particular, the significance of the relation (2.12).

The basic relationships used in the following are two expressions for a particle (baryon) magnetic moment. Identifying the magnitude of a baryon moment expressed in terms of its spin, **S**, and g-factor, G, with the magnitude of the sum of the moments of its constituent quarks (from (2.16))

$$\mu = e(g_3/2m_3)K = e(G/2M)S \tag{3.1}$$

From (3.1), defining

$$\gamma = (G/g_3)(S/K) \tag{3.2a}$$

$$M = m_3\gamma \tag{3.2b}$$

Hypothesising the existence of a particle of mass M^* with a quark substructure similar to that of the particle of mass M (the relationships between these particles will be made more precise in the following), and, distinguishing the properties associated with this new particle from those associated with the particle of mass M by means of an appended asterisk, the magnetic moment of the new particle would satisfy

$$\mu^* = e(g_3^*/2m_3^*)K^* = e(G^*/2M^*)S^* \tag{3.3}$$

From (3.3), defining

$$\gamma^* = (G^*/g_3^*)S^*/K^* \tag{3.4a}$$

$$M^* = m_3^*\gamma^* \tag{3.4b}$$

From (3.2) and (3.4)

$$M^*/M = (\gamma^*/\gamma)m_3^*/m_3 \tag{3.5}$$

Further characterising the relationship between the particles of mass M^* and M in terms of a parameter f defined by

$$m_1 = fm_3^* \tag{3.6}$$

and using (3.5),

$$M^*/M = m_1/m_3(\gamma^*/f\gamma) \tag{3.7}$$

If the parameters α from (2.12) and f from (3.6) satisfy

$$\alpha f = \gamma^*/\gamma \tag{3.8a}$$

and if

$$M^*/M = M/m \tag{3.8b}$$

then (3.7) is identical with (2.13). From this point of view the right-hand side of (2.13) would correspond basically to an expression for the mass ratio M^*/M; because of the conditions (3.8) it would also correspond to an expression for M/m. This idea was utilised in conjunction with the previous models, where it was further speculated that m, M and M^* correspond to the masses of the first three particles in an (infinite) sequence of particles; specifically (3.8b) was generalised to

$$M_{n+1}/M_n = M_2/M_1 = M/m \tag{3.9}$$

where n would denote the position of a particle in the postulated sequence. The masses of the particles in the sequence would be $M_{n+1}/M_1 = (M/m)^n$.

In the following a convenient formalism for the description of such sequences of particles will be developed; the collocation of the relation (2.12) within this formalism is evidenced and, with reference to the model discussed in the previous section, a criterion by which the length of a particle sequence may be fixed is obtained and a specific particle sequence is examined in detail.

Adjoining an additional subscript to the symbols which have previously denoted the properties associated with a particle, where this new subscript identifies the position of a particle in a presumed sequence of particles, the relations (3.2), (3.6) and (2.12) are respectively generalised to

$$\gamma_n = (G_n/g_{3,n})S_n/K_n \tag{3.10a}$$

$$M_n = \gamma_n m_{3,n} \tag{3.10b}$$

$$m_{1,n} = f_n m_{3,n+1} \tag{3.11}$$

$$M_{n+1}/M_n = \alpha_n m_{1,n+1}/m_{3,n+1} \tag{3.12}$$

It is noteworthy that α_n and f_n are not properties of a particle but relate successive particles in the sequence. Defining new parameters, ρ_n and η_n, which respectively relate the three quark masses $(m_{3,n-1}, m_{3,n}, m_{3,n+1})$ and the three particle masses (M_{n-1}, M_n, M_{n+1}) by

$$m_{3,n+1}/m_{3,n} = (m_{3,n}/m_{3,n-1})\rho_n = m_{3,2}/m_{3,1}\Pi_{k=2}^{n}\rho_k \tag{3.13}$$

$$\gamma_{n+1}/\gamma_n = (\gamma_n/\gamma_{n-1})\eta_n = \gamma_2/\gamma_1\Pi_{k=2}^{n}\eta_k \tag{3.14}$$

and using also (3.10), (3.11) and (3.12),

$$\gamma_{n+1}/\gamma_n = \alpha_n f_{n+1}\rho_n \tag{3.15}$$

From (3.10), (3.13) and (3.14),

$$\begin{aligned} M_{n+1}/M_n &= (\gamma_{n+1}/\gamma_n)m_{3,n+1}/m_{3,n} \\ &= M_2/M_1\Pi_{k=2}^{n}\eta_k\rho_k \end{aligned} \tag{3.16}$$

and, defining

$$X(n) = \Pi_{k=2}^{n}\Pi_{i=2}^{k}\eta_i \tag{3.17}$$

and using (3.10) and (3.14),

$$M_{n+1}/m_{3,n+1} = (M_1/m_{3,1})(\gamma_2/\gamma_1)^n X(n) \tag{3.18}$$

In the following, only a particle sequence whose first two members are the electron and proton $(M_1 = m, M_2 = M(P))$ will be considered. By generalising some of the results obtained for the proton in the previous section, a condition limiting the length of the 'proton sequence' may be obtained from (3.18); in particular, using proton parameter values from (2.23)

$$g_{3,n} = 2 \tag{3.19a}$$

$$K_n/S_n \cong K_{0,n}/S_n = 1 \tag{3.19b}$$

and, generalising (2.9a, d),

$$W_n R_n = d = 1 \tag{3.19c}$$

$$m_{3,n} R_n = b_{3,n} = 1 \tag{3.19d}$$

From (3.10) and (3.19),

$$\gamma_n \cong G_n/2 \tag{3.20a}$$

$$W_n = m_{3,n} \tag{3.20b}$$

and, in particular,

$$M_1/W_1 = M_1/m_{3,1} = \gamma_1 \cong G_1/2 \cong 1 \tag{3.20c}$$

the mass of the electron would (almost) equal its electromagnetic self-energy.

Using (3.20), (3.18) becomes

$$M_{n+1}/W_{n+1} = (M_1/W_1)(\gamma_2/\gamma_1)^n X(n) \tag{3.21a}$$

$$= \gamma_1(\gamma_2/\gamma_1)^n X(n) \tag{3.21b}$$

$$= (G_2/2)^n X(n) \tag{3.21c}$$

Obviously M_n and W_n must obey the condition

$$M_n/W_n \geqq 1 \tag{3.22a}$$

thus, from (3.21), if the sequence η_n is such that

$$M_N/W_N = (G_2/2)^{N-1} X(N-1) \geqq 1 \tag{3.22b}$$

$$M_{N+1}/W_{N+1} = (G_2/2)^N X(N) < 1 \tag{3.22c}$$

then the proton sequence will terminate at its Nth member.

It may be observed that (3.16) would be equivalent to (3.9) if, for example, $\eta_k = 1/\rho_k$. However in the following a different sequence, having relatively more interesting properties, will be considered.

Hypothesising

$$\eta_n = n-1/n \tag{3.23}$$

and using (3.17),

$$X(n) = 1/n! \tag{3.24}$$

Using (3.24) and the measured value of the proton g-factor $G_2/2 = 2{\cdot}8$, $M_6/W_6 = 43/30, M_7/W_7 = 241/360$, and thus the proton sequence would contain N=6 members. If M_6/W_6 had turned out to be very close to one, it would be possible to characterise the limit of the sequence M_n by the precise constraint $M_N/W_N = 1$, which would then lead to a prediction for G_2; by considering also the ρ_n a precise constraint on the sequence limit may be formulated.

Hypothesising that the ρ_n are independent of n, all equal a constant ρ, using (3.16) and (3.23),

$$M_{n+1}/M_n = (M_2/M_1)\rho^{n-1}/n \tag{3.25a}$$

$$M_{n+1}/M_1 = (M_2/M_1)^n \rho^{n(n-1)/2}/n! \tag{3.25b}$$

and using also (3.21), (3.24) and (3.20)

$$W_{n+1}/W_1 = (M_2/M_1)^n \rho^{n(n-1)/2}/(G_2/2)^n \tag{3.25c}$$

From (3.25b, c) it may be observed that for $3M_2/M_1 \lesssim \rho < 1$, the sequences M_n and W_n would have maxima for $n > 2$; if ρ were sufficiently small the maxima would occur for small n. This suggests the possibility of refining the condition limiting the length of the proton sequence as follows: the value of ρ is such that the sequence M_n has a maximum 'at' the limit determined by the sequence η_n; precisely the sequence limiting condition is

$$M_7 = M_6 > M_5 \tag{3.26a}$$

$$M_6 = M_7 > M_8 \tag{3.26b}$$

$$W_7 > M_6 > W_6 \tag{3.26c}$$

that is, the condition (3.22a) is violated because M_n stops increasing at M_6 whereas the sequence W_n continues to increase to $W_7 > M_6$. From (3.26) and (3.25a),

$$M_7/M_6 = (M_2/M_1)\rho^5/6 = 1$$

$$M_2/M_1 = 6\rho^{-5} \tag{3.27}$$

thus ρ may be determined in terms of the proton–electron mass ratio.

At this point the obvious identification for ρ is

$$1/\rho = \pi = 3{\cdot}14159\ldots \tag{3.28}$$

since, in this case,

$$M_2/M_1 = 6\pi^5 = 1836{\cdot}1181 \tag{3.29a}$$

$$M_2 = 938{\cdot}2639 \text{ MeV} \tag{3.29b}$$

The expression (3.29a) for the proton–electron mass ratio has been suggested previously by several authors (Lenz, 1951; Good, 1970; Wyler, 1971).

The condition (3.22a) limiting the length of the proton sequence and the expression (3.13) defining ρ can also be given spatial or geometrical interpretations. Using (3.19c), (3.22a) may be expressed as $M_nR_n \geqq 1$; the radii of the particles in the proton sequence must be greater than (or equal to) their reduced Compton wavelengths. Using (3.19b) and (3.28), (3.13) corresponds to $\pi R_n = R_{n-1} R_{n+1}$; the cross-sectional area of the nth member of the sequence equals the area of the rectangle formed from the radii of the $(n-1)$th and $(n+1)$th members.

Returning finally to the initial objectives of this section, it may be noted that the parameters α_n and f_n may be separately related to the η_n and ρ_n if it is assumed that

$$m_{1,n}/m_{3,n} = F(P) \tag{3.30}$$

where $F(P)$ is a constant, independent of n. In this case the $m_{1,n}$ will also be related by an expression identical with (3.13); using (3.11) and (3.13),

$$f_{n+1}/f_n = 1/\rho_{n+1} = \pi \tag{3.31}$$

From (3.14), (3.15) and (3.31), $\alpha_{n+1}/\alpha_n = \eta_{n+1}\rho_{n+1}$ and $\gamma_{n+1}/\gamma_n = \alpha_n f_n$. With (3.30), (3.14) becomes $M_{n+1}/M_n = \alpha_n F(P)$. In particular, using the value $\alpha_1 = 15/2$ suggested in the previous section, $\gamma_2/\gamma_1 = 15/2\, f_1$ and $M_2/M_1 = 15/2F(P)$.

4. *Summary*

The general conclusions suggested by the present model are qualitatively similar to those of the previous models. Two types of baryon substructure are implied, one being characterised by a large spatial extension (R_3=R) and a small associated quark mass ($m_3 < M(P)$), the other being characterised by a very small spatial extension ($R_1 \ll R$) and a very large associated quark mass ($m_1 \gg M(P)$).

The present parameterisation is simpler than those previously considered in that it considers only one pair of quark masses, rather then allowing a different pair for each particle; this approach leads to the possibility of determining the parameters associated with all baryons in terms of the parameters associated with the proton.

In Section 2, fairly accurate values for the ratios of the baryon magnetic moments to the proton moment are obtained, together with an accurate value for the proton mass. This proton mass value results from especially simple identifications for the proton parameters but the significance of the relation (2.12) leading to the expression (2.15) for the proton mass is obscure and the value 15/2 for the parameter $\alpha(P)$ would seem to require explanation.

In Section 3 a possible explanation of (2.12) in terms of a presumed particle sequence, M_n, whose first two members are the electron and proton was presented. This approach necessitated the generalisation of $\alpha(P)$ to the sequence $\alpha_n (\alpha_1 = \alpha(P))$ and the introduction of also another parameter sequence f_n. The sequence M_n could also be characterised in terms of parameter sequences ρ_n and η_n whose elements could be fixed such as to obtain a finite length for the sequence M_n and a second prediction (3.29) for the proton-electron mass ratio. The α_n and f_n could be related to the ρ_n and η_n but essentially because the ρ_n and η_n relate properties associated with three successive members of the sequence whereas α_n and f_n relate properties associated with two successive members, only the ratios f_{n+1}/f_n and α_{n+1}/α_n

could be determined leaving the values of f_1 and $\alpha_1 = \alpha(P)$ theoretically undetermined

The expressions (2.23e) and (3.29a) both yield estimates for the proton-electron mass ratio which agree within experimental error with the measured values for this quantity However, combining these expressions the fine structure constant may be predicted to obey

$$1/e^2 = 4\pi^5/9 + 37/36 = 137{\cdot}03653$$

The deviation of this result from the measured value $1/e^2 = 137{\cdot}03602 \pm {\cdot}00021$ suggests that either (2.23e) or (3.29a) is slightly inaccurate (or both are).

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